1.4 Measure and Classify Angles

Before You named and measured line segments.

Now You will name, measure, and classify angles.

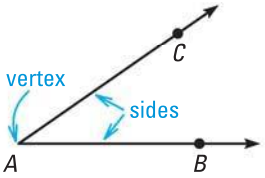
Why? So you can identify congruent angles, as in Example 4.

Key Vocabulary

- **angle**
  acute, right, obtuse, straight
- **sides, vertex of an angle**
- **measure of an angle**
- **congruent angles**
- **angle bisector**

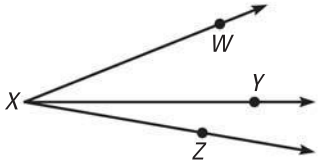
An **angle** consists of two different rays with the same endpoint. The rays are the **sides** of the angle. The endpoint is the **vertex** of the angle.

The angle with sides $\overrightarrow{AB}$ and $\overrightarrow{AC}$ can be named $\angle BAC$, $\angle CAB$, or $\angle A$. Point A is the vertex of the angle.



EXAMPLE 1 Name angles

Name the three angles in the diagram.



$\angle WXY$, or $\angle YXW$

$\angle YXZ$, or $\angle ZXY$

$\angle WXZ$, or $\angle ZXW$

You should not name any of these angles $\angle X$ because all three angles have X as their vertex.

MEASURING ANGLES A protractor can be used to approximate the *measure* of an angle. An angle is measured in units called *degrees* (°). For instance, the measure of $\angle WXZ$ in Example 1 above is 32°. You can write this statement in two ways.

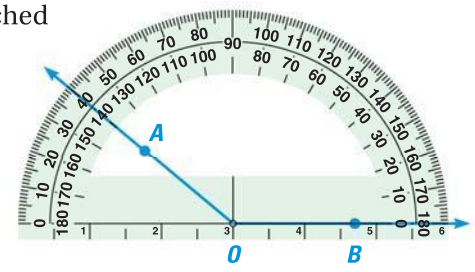
Words The measure of $\angle WXZ$ is 32°.

Symbols $m\angle WXZ = 32°$

POSTULATE *For Your Notebook*

POSTULATE 3 Protractor Postulate

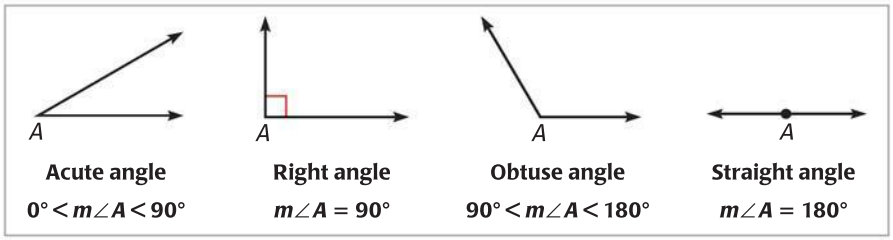
Consider $\overrightarrow{OB}$ and a point A on one side of $\overrightarrow{OB}$. The rays of the form $\overrightarrow{OA}$ can be matched one to one with the real numbers from 0 to 180.



The **measure** of $\angle AOB$ is equal to the absolute value of the difference between the real numbers for $\overrightarrow{OA}$ and $\overrightarrow{OB}$.

CLASSIFYING ANGLES Angles can be classified as **acute**, **right**, **obtuse**, and **straight**, as shown below.

READ DIAGRAMS
A red square inside an angle indicates that the angle is a right angle.

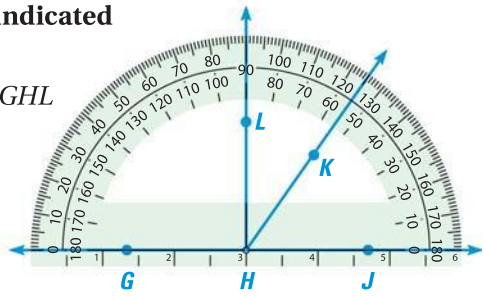


Acute angle	Right angle	Obtuse angle	Straight angle
$0° < m\angle A < 90°$	$m\angle A = 90°$	$90° < m\angle A < 180°$	$m\angle A = 180°$

EXAMPLE 2 Measure and classify angles

Use the diagram to find the measure of the indicated angle. Then classify the angle.

a. $\angle KHJ$ **b.** $\angle GHK$ **c.** $\angle GHJ$ **d.** $\angle GHL$



Solution

A protractor has an inner and an outer scale. When you measure an angle, check to see which scale to use.

a. $\overrightarrow{HJ}$ is lined up with the 0° on the inner scale of the protractor. $\overrightarrow{HK}$ passes through 55° on the inner scale. So, $m\angle KHJ = 55°$. It is an acute angle.

b. $\overrightarrow{HG}$ is lined up with the 0° on the outer scale, and $\overrightarrow{HK}$ passes through 125° on the outer scale. So, $m\angle GHK = 125°$. It is an obtuse angle.

c. $m\angle GHJ = 180°$. It is a straight angle.

d. $m\angle GHL = 90°$. It is a right angle.

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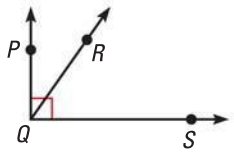
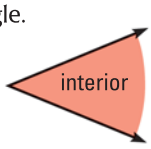
GUIDED PRACTICE for Examples 1 and 2

1. Name all the angles in the diagram at the right. Which angle is a right angle?

2. Draw a pair of opposite rays. What type of angle do the rays form?



READ DIAGRAMS
A point is in the *interior* of an angle if it is between points that lie on each side of the angle.

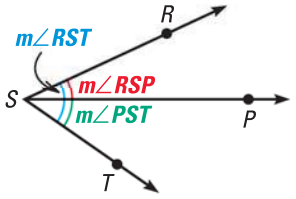


POSTULATE *For Your Notebook*

POSTULATE 4 Angle Addition Postulate

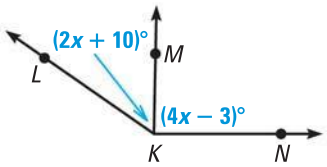
Words If P is in the interior of $\angle RST$, then the measure of $\angle RST$ is equal to the sum of the measures of $\angle RSP$ and $\angle PST$.

Symbols If P is in the interior of $\angle RST$, then $m\angle RST = m\angle RSP + m\angle PST$.



EXAMPLE 3 Find angle measures

ALGEBRA Given that $m\angle LKN = 145°$, find $m\angle LKM$ and $m\angle MKN$.



Solution

STEP 1 **Write** and solve an equation to find the value of x.

$m\angle LKN = m\angle LKM + m\angle MKN$	Angle Addition Postulate
$145° = (2x + 10)° + (4x - 3)°$	Substitute angle measures.
$145 = 6x + 7$	Combine like terms.
$138 = 6x$	Subtract 7 from each side.
$23 = x$	Divide each side by 6.

STEP 2 **Evaluate** the given expressions when $x = 23$.

$m\angle LKM = (2x + 10)° = (2 \cdot 23 + 10)° = 56°$

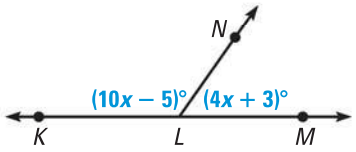
$m\angle MKN = (4x - 3)° = (4 \cdot 23 - 3)° = 89°$

▶ So, $m\angle LKM = 56°$ and $m\angle MKN = 89°$.

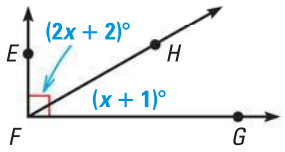
GUIDED PRACTICE for Example 3

Find the indicated angle measures.

3. Given that $\angle KLM$ is a straight angle, find $m\angle KLN$ and $m\angle NLM$.

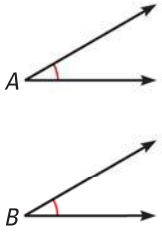


4. Given that $\angle EFG$ is a right angle, find $m\angle EFH$ and $m\angle HFG$.



CONGRUENT ANGLES Two angles are **congruent angles** if they have the same measure. In the diagram below, you can say that "the measure of angle A is equal to the measure of angle B," or you can say "angle A *is congruent to* angle B."

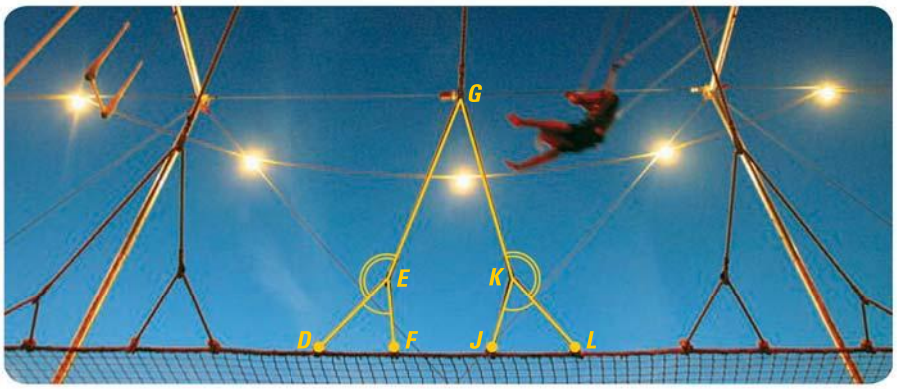
READ DIAGRAMS
Matching arcs are used to show that angles are congruent. If more than one pair of angles are congruent, double arcs are used, as in Example 4 on page 27.



Angle measures are equal.	Angles are congruent.
$m\angle A = m\angle B$	$\angle A \cong \angle B$
"is equal to"	"is congruent to"

EXAMPLE 4 Identify congruent angles

TRAPEZE The photograph shows some of the angles formed by the ropes in a trapeze apparatus. Identify the congruent angles. If $m\angle DEG = 157°$, what is $m\angle GKL$?



Solution

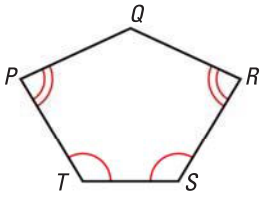
There are two pairs of congruent angles:

$\angle DEF \cong \angle JKL$ and $\angle DEG \cong \angle GKL$.

Because $\angle DEG \cong \angle GKL$, $m\angle DEG = m\angle GKL$. So, $m\angle GKL = 157°$.

GUIDED PRACTICE for Example 4

Use the diagram shown at the right.

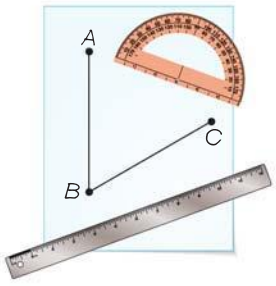


5. Identify all pairs of congruent angles in the diagram.

6. In the diagram, $m\angle PQR = 130°$, $m\angle QRS = 84°$, and $m\angle TSR = 121°$. Find the other angle measures in the diagram.

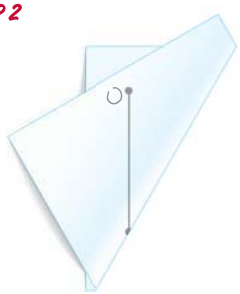
ACTIVITY FOLD AN ANGLE BISECTOR

STEP 1



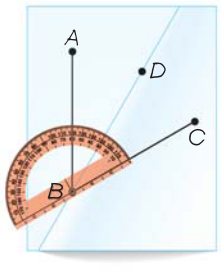
Use a straightedge to draw and label an acute angle, $\angle ABC$.

STEP 2



Fold the paper so that $\overrightarrow{BC}$ is on top of $\overrightarrow{BA}$.

STEP 3

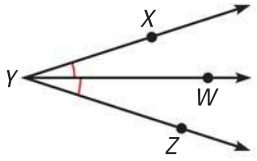


Draw a point D on the fold inside $\angle ABC$. Then measure $\angle ABD$, $\angle DBC$, and $\angle ABC$. What do you observe?

An **angle bisector** is a ray that divides an angle into two angles that are congruent. In the activity on page 27, $\overrightarrow{BD}$ bisects $\angle ABC$. So, $\angle ABD \cong \angle DBC$ and $m\angle ABD = m\angle DBC$.

EXAMPLE 5 Double an angle measure

In the diagram at the right, $\overrightarrow{YW}$ bisects $\angle XYZ$, and $m\angle XYW = 18°$. Find $m\angle XYZ$.



Solution

By the Angle Addition Postulate, $m\angle XYZ = m\angle XYW + m\angle WYZ$. Because $\overrightarrow{YW}$ bisects $\angle XYZ$, you know that $\angle XYW \cong \angle WYZ$.

So, $m\angle XYW = m\angle WYZ$, and you can write

$$m\angle XYZ = m\angle XYW + m\angle WYZ = 18° + 18° = 36°.$$

GUIDED PRACTICE for Example 5

7. Angle MNP is a straight angle, and $\overrightarrow{NQ}$ bisects $\angle MNP$. Draw $\angle MNP$ and $\overrightarrow{NQ}$. Use arcs to mark the congruent angles in your diagram, and give the angle measures of these congruent angles.

1.4 EXERCISES

HOMEWORK KEY

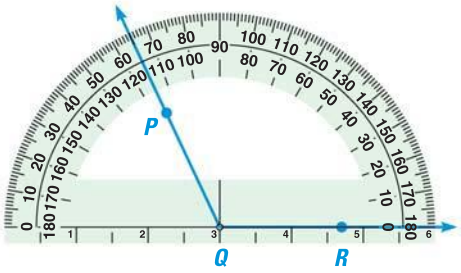
○ = **WORKED-OUT SOLUTIONS** on p. WS1 for Exs. 15, 23, and 53

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 21, 27, 43, and 62

SKILL PRACTICE

1. VOCABULARY Sketch an example of each of the following types of angles: acute, obtuse, right, and straight.

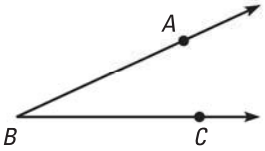
2. ★ WRITING *Explain* how to find the measure of $\angle PQR$, shown at the right.



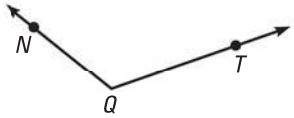
EXAMPLE 1 on p. 24 for Exs. 3–6

NAMING ANGLES AND ANGLE PARTS **In Exercises 3–5, write three names for the angle shown. Then name the vertex and sides of the angle.**

3.

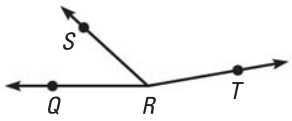


4.



5.

6. NAMING ANGLES Name three different angles in the diagram at the right.

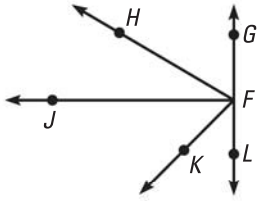


EXAMPLE 2
on p. 25
for Exs. 7–21

CLASSIFYING ANGLES **Classify the angle with the given measure as *acute*, *obtuse*, *right*, or *straight*.**

7. $m\angle W = 180°$ **8.** $m\angle X = 30°$ **9.** $m\angle Y = 90°$ **10.** $m\angle Z = 95°$

MEASURING ANGLES **Trace the diagram and extend the rays. Use a protractor to find the measure of the given angle. Then classify the angle as *acute*, *obtuse*, *right*, or *straight*.**



11. $\angle JFL$ **12.** $\angle GFH$

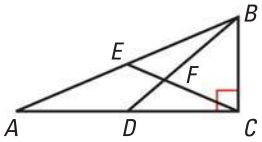
13. $\angle GFK$ **14.** $\angle GFL$

NAMING AND CLASSIFYING **Give another name for the angle in the diagram below. Tell whether the angle appears to be *acute*, *obtuse*, *right*, or *straight*.**

15. $\angle ACB$ **16.** $\angle ABC$

17. $\angle BFD$ **18.** $\angle AEC$

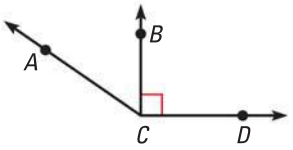
19. $\angle BDC$ **20.** $\angle BEC$



21. ★ MULTIPLE CHOICE Which is a correct name for the obtuse angle in the diagram?

Ⓐ $\angle ACB$ Ⓑ $\angle ACD$

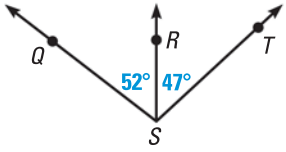
Ⓒ $\angle BCD$ Ⓓ $\angle C$

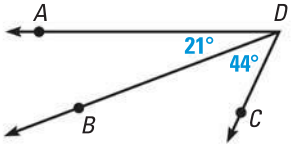


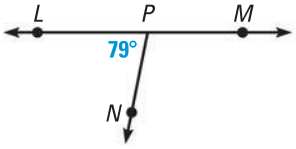
EXAMPLE 3
on p. 26
for Exs. 22–27

ANGLE ADDITION POSTULATE **Find the indicated angle measure.**

22. $m\angle QST =$? **23.** $m\angle ADC =$? **24.** $m\angle NPM =$?

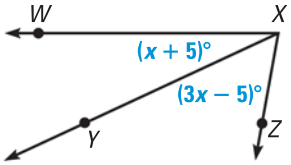




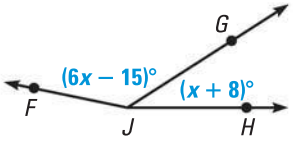


xy ALGEBRA **Use the given information to find the indicated angle measure.**

25. Given $m\angle WXZ = 80°$, find $m\angle YXZ$.



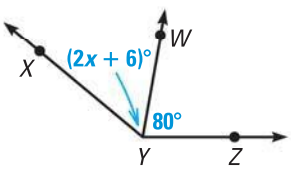
26. Given $m\angle FJH = 168°$, find $m\angle FJG$.



27. ★ MULTIPLE CHOICE In the diagram, the measure of $\angle XYZ$ is 140°. What is the value of x?

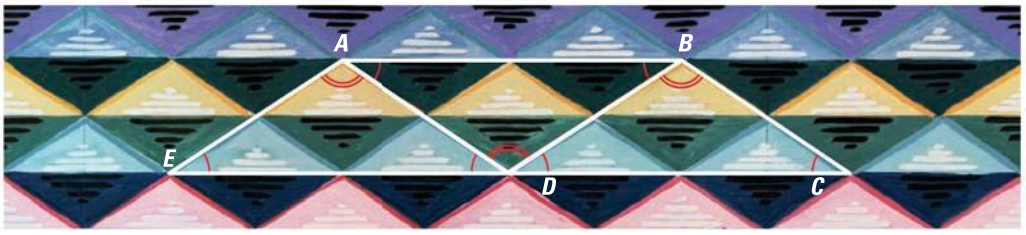
Ⓐ 27 Ⓑ 33

Ⓒ 67 Ⓓ 73



EXAMPLE 4
on p. 27
for Ex. 28

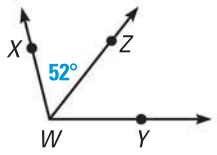
28. CONGRUENT ANGLES In the photograph below, $m\angle AED = 34°$ and $m\angle EAD = 112°$. Identify the congruent angles in the diagram. Then find $m\angle BDC$ and $m\angle ADB$.



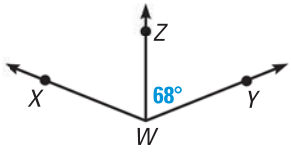
EXAMPLE 5
on p. 28
for Exs. 29–32

ANGLE BISECTORS **Given that $\overrightarrow{WZ}$ bisects $\angle XWY$, find the two angle measures not given in the diagram.**

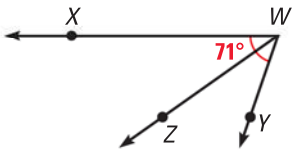
29.



30.



31.



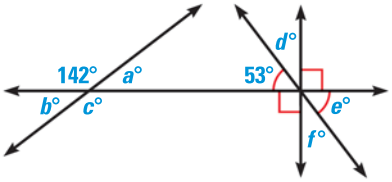
32. ERROR ANALYSIS $\overrightarrow{KM}$ bisects $\angle JKL$ and $m\angle JKM = 30°$. *Describe* and correct the error made in stating that $m\angle JKL = 15°$. Draw a sketch to support your answer.

FINDING ANGLE MEASURES **Find the indicated angle measure.**

33. $a°$ **34.** $b°$

35. $c°$ **36.** $d°$

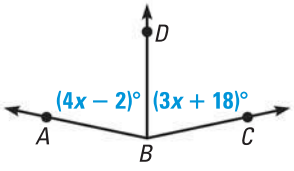
37. $e°$ **38.** $f°$



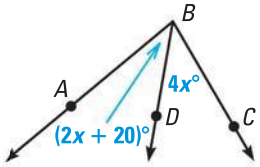
39. ERROR ANALYSIS A student states that $\overrightarrow{AD}$ can bisect $\angle AGC$. *Describe* and correct the student's error. Draw a sketch to support your answer.

ALGEBRA **In each diagram, $\overrightarrow{BD}$ bisects $\angle ABC$. Find $m\angle ABC$.**

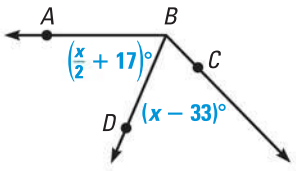
40.



41.



42.



43. ★ SHORT RESPONSE You are measuring $\angle PQR$ with a protractor. When you line up $\overrightarrow{QR}$ with the 20° mark, $\overrightarrow{QP}$ lines up with the 80° mark. Then you move the protractor so that $\overrightarrow{QR}$ lines up with the 15° mark. What mark does $\overrightarrow{QP}$ line up with? *Explain*.

ALGEBRA **Plot the points in a coordinate plane and draw $\angle ABC$. Classify the angle. Then give the coordinates of a point that lies in the interior of the angle.**

44. $A(3, 3), B(0, 0), C(3, 0)$

45. $A(-5, 4), B(1, 4), C(-2, -2)$

46. $A(-5, 2), B(-2, -2), C(4, -3)$

47. $A(-3, -1), B(2, 1), C(6, -2)$

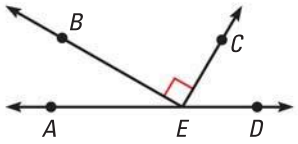
○ = WORKED-OUT SOLUTIONS on p. WS1

★ = STANDARDIZED TEST PRACTICE

48. **ALGEBRA** Let $(2x - 12)°$ represent the measure of an acute angle. What are the possible values of x?

49. **CHALLENGE** $\overrightarrow{SQ}$ bisects $\angle RST$, $\overrightarrow{SP}$ bisects $\angle RSQ$, and $\overrightarrow{SV}$ bisects $\angle RSP$. The measure of $\angle VSP$ is 17°. Find $m\angle TSQ$. *Explain*.

50. **FINDING MEASURES** In the diagram, $m\angle AEB = \frac{1}{2} \cdot m\angle CED$, and $\angle AED$ is a straight angle. Find $m\angle AEB$ and $m\angle CED$.



PROBLEM SOLVING

51. **SCULPTURE** In the sculpture shown in the photograph, suppose the measure of $\angle LMN$ is 79° and the measure of $\angle PMN$ is 47°. What is the measure of $\angle LMP$?

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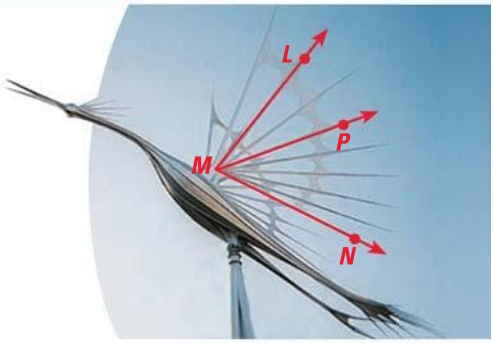
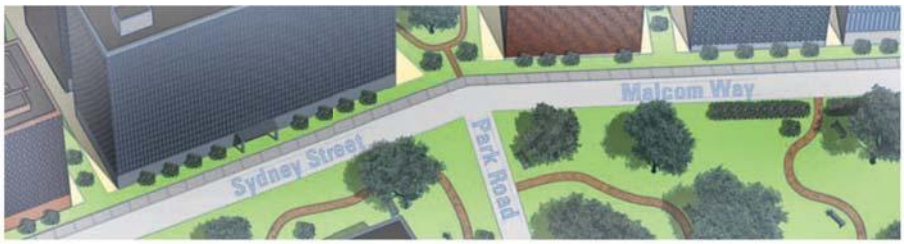


52. **MAP** The map shows the intersection of three roads. Malcom Way intersects Sydney Street at an angle of 162°. Park Road intersects Sydney Street at an angle of 87°. Find the angle at which Malcom Way intersects Park Road.



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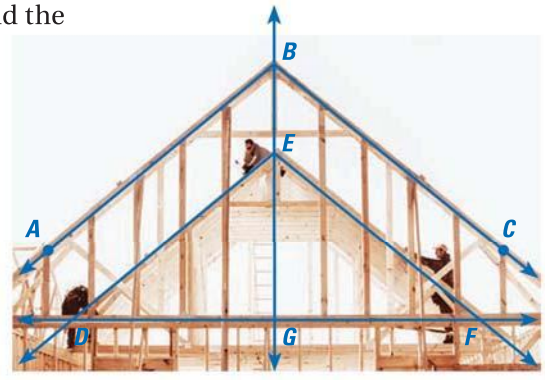
EXAMPLES 4 and 5 on pp. 27–28 for Exs. 53–55

CONSTRUCTION **In Exercises 53–55, use the photograph of a roof truss.**

53. In the roof truss, $\overrightarrow{BG}$ bisects $\angle ABC$ and $\angle DEF$, $m\angle ABC = 112°$, and $\angle ABC \cong \angle DEF$. Find the measure of the following angles.

a. $m\angle DEF$ **b.** $m\angle ABG$

c. $m\angle CBG$ **d.** $m\angle DEG$



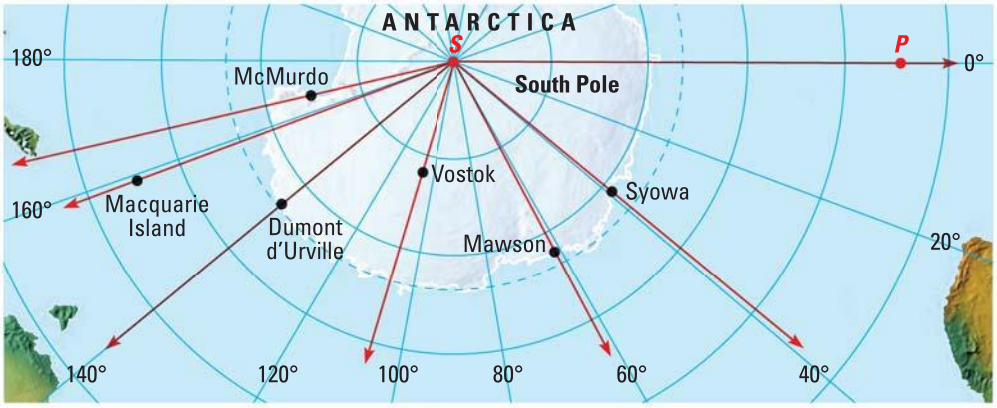
54. In the roof truss, $\overrightarrow{GB}$ bisects $\angle DGF$. Find $m\angle DGE$ and $m\angle FGE$.

55. Name an example of each of the following types of angles: *acute*, *obtuse*, *right*, and *straight*.

GEOGRAPHY **For the given location on the map, estimate the measure of $\angle PSL$, where P is on the Prime Meridian (0° longitude), S is the South Pole, and L is the location of the indicated research station.**

56. Macquarie Island **57.** Dumont d'Urville **58.** McMurdo

59. Mawson **60.** Syowa **61.** Vostok

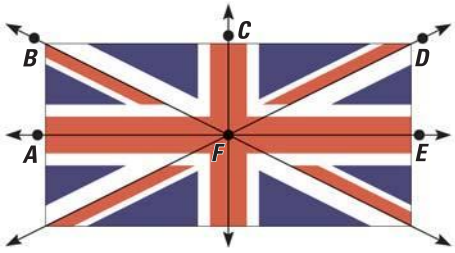


62. ★ **EXTENDED RESPONSE** In the flag shown, $\angle AFE$ is a straight angle and $\overrightarrow{FC}$ bisects $\angle AFE$ and $\angle BFD$.

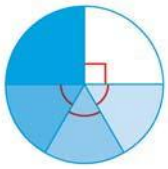
a. Which angles are acute? obtuse? right?

b. Identify the congruent angles.

c. If $m\angle AFB = 26°$, find $m\angle DFE$, $m\angle BFC$, $m\angle CFD$, $m\angle AFC$, $m\angle AFD$, and $m\angle BFD$. *Explain.*



63. **CHALLENGE** Create a set of data that could be represented by the circle graph at the right. *Explain* your reasoning.



MIXED REVIEW

PREVIEW
Prepare for Lesson 1.5 in Ex. 64.

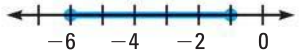
64. You and a friend go out to dinner and each pay for your own meal. The total cost of the two meals is $25. Your meal cost $4 more than your friend's meal. How much does each meal cost? *(p. 894)*

Graph the inequality on a number line. Tell whether the graph is a *segment*, a *ray* or *rays*, a *point*, or a *line*. *(p. 2)*

65. $x \le -8$ **66.** $x \ge 6$ **67.** $-3 \le x \le 5$

68. $x \ge -7$ and $x \le -1$ **69.** $x \ge -2$ or $x \le 4$ **70.** $|x| \ge 0$

Find the coordinate of the midpoint of the segment. *(p. 15)*

71. 

72. 

73.